Solution: where two curves intersect

7.1 Notes: Linear and Nonlinear Systems of Equations

Ex: 1 Solve the system of equations by **substitution.**

$x - y = 3$ → solve for x $x = (3 + y)$

$3x - 4y = 7$

$3(3+y) - 4y = 7$

$9 + 3y - 4y = 7$

$9 - y = 7$

$-y = -2$

$y = 2$

$x - 2 = 3$

$x = 5$

Soln: $(5, 2)$

Ex: 2 Solve the system of equations by **substitution.**

$3x^2 - 2x + y = 0$ → $y = -3x^2 + 2x$

$-x + y = -2$ → $y = x - 2$

$-3x^2 + 2x = 1x - 2$

$+3x^2 - 2x \quad +3x^2 - 2x$

$0 = 3x^2 - x - 2$

$0 = (3x + 2)(x - 1)$

$3x + 2 = 0 \qquad x - 1 = 0$

SolN:

$\left(-\frac{2}{3}, \frac{-8}{3}\right)$ & $(1, -1)$

$y = -\frac{2}{3} - 2$

$-\frac{2}{3} - \frac{6}{3}$

$y = 1 - 2$

Ex: 3 Solve the system of equations by **substitution.**

$3x - y = -1 \qquad y = 3x + 1$

$5x^2 + y = 0$ → $y = -5x^2$

$3x - (-5x^2) = -1$

$5x^2 + 3x + 1 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(5)(1)}}{10}$

$\frac{-3 \pm \sqrt{-11}}{10}$ ← non. real ans.

$\boxed{\text{No solution}}$

Ex: 4 Solve the system of equations by **substitution.**

$\begin{cases} y = (4x - 4) \\ 8x - 2y = 8 \end{cases}$

$8x - 2(4x - 4) = 8$

$\cancel{8x} - \cancel{8x} + 8 = 8$

$8 = 8$ True

$0 = 0$

Soln.

$\boxed{\text{Infinitely many solutions}}$

Scanned by CamScanner

Solve each of the following system of equations by **elimination.**

Ex: 5 $\begin{cases} 3x + 2y = 4 \\ 5x - 2y = 12 \end{cases}$

+

$8x = 16$

$x = 2$

$3(2) + 2y = 4$

$6 + 2y = 4$

$2y = -2$

$y = -1$

$\boxed{\text{Soln: } (2,-1)}$

Ex: 6 $3\begin{cases} 2x + y = 1 \\ 6x - 3y = 6 \end{cases}$

$6x + 3y = 3$

$6x - 3y = 6$

+

$\frac{12x}{12} = \frac{9}{12}$

$x = \frac{3}{4}$

$\frac{2}{1}\left(\frac{3}{4}\right) + y = 1$

$\frac{3}{2} + y = 1\frac{2}{2}$

$-\frac{3}{2} \qquad -\frac{3}{2}$

$y = -\frac{1}{2}$

$\boxed{\text{Soln: } \left(\frac{3}{4}, -\frac{1}{2}\right)}$

Ex: 7 $\begin{cases} 3x + 2y = 7 \\ 2x + 5y = 1 \end{cases}$

$(3,-1)$

Ex: 8

$-3\begin{cases} x + 2y = 4 \\ 3x + 6y = 13 \end{cases} \rightarrow$

$-3x - 6y = -12$

$+ \quad 3x + 6y = 13$

$0 = 1$ FALSE

No Soln.

Ex: 9 The weekly ticket sales for a new animated movie decreased each week. At the same time, the weekly ticket sales for a new horror movie increased each week. Models that approximate the weekly ticket sales S (in millions of dollars) for the movies are

$S = 108 - 9.4x$

$S = 16 + 9x$

where x represents the number of weeks each movie was in theaters, with $x = 0$ corresponding to the opening weekend. After how many weeks will the ticket sales for the two movies be equal? What is the weekly ticket sales after the # of wks where the 2 movies will be equal?

$108 - 9.4x = 16 + 9x$

$92 = 18.4x$

$5 = x$

after 5 weeks, ticket sales will be equal

$16 + 9(5) = 61 \rightarrow \$61,000,000$

Ex: 10 Finding a Break-Even Point

A company that manufactures running shoes has a fixed cost of $300,000. Additionally, it costs $30 to produce each pair of shoes. They are sold at $80 per pair.

a. Write the cost function, C, of producing x pairs of running shoes.

$C(x) = 30x + 300\,000$

b. Write the revenue function, R, from the sale of x pairs of running shoes.

$R(x) = 80x$

c. Determine the break-even point. Describe what this means.

$C(x) = R(x)$

$30x + 300\,000 = 80x$

$300\,000 = 50x$

$6000 = x$

break even pt $\boxed{(6000, 480\,000)}$

$80(6000) = 480\,000$

$30(6000) + 300\,000 = \uparrow$

- when 6000 pairs of shoes are produced and sold, the cost & revenue are the same @ $480,000

Determining the Equilibrium Price

Supply Curve: As one variable increases, the other also increases. (Suppliers will increase production if they can get higher prices for their product)

Demand Curve: As one variable increases, the other decreases. (Demand for a product by consumers will decrease as the price goes up)

A point where the supply and demand curve intersect is an equilibrium point. The corresponding price is the equilibrium price.

Ex: 11

Nibok Manufacturing has determined that production and price of a new tennis shoe should be geared to the equilibrium point for this system of equations.

$p = 160 - 5x$ demand
$p = 35 + 20x$ Supply

The price, p, is in dollars and the number of shoes, x, is in millions of pairs. Find the equilibrium point.

$160 - 5x = 35 + 20x$

$125 = 25x$

$5 = x$

(million pairs of shoes)

$(5, 135)$

Equil. price $35 + 20(5)$

$\hookrightarrow = \$135.00$ per pair